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Abstract

An approach for design and generation of low-noise helical gears with localized bearing contact is proposed. The approach is applied to double circular arc helical gears and modified involute helical gears. The reduction of noise and vibration is achieved by application of a predesigned parabolic function of transmission errors that is able to absorb a discontinuous linear function of transmission errors caused by misalignment. The localization of the bearing contact is achieved by the mismatch of pinion-gear tooth surfaces. Computerized simulation of meshing and contact of the designed gears demonstrated that the proposed approach will produce a pair of gears that has a parabolic transmission error function even when misalignment is present. Numerical examples for illustration of the developed approach are given.

NOMENCLATURE

a	Parabola parameter
b	Slope of linear function
E_{pg}	Shortest distance between pinion-gear
L_{ij}	Line of tangency between surfaces Σ_i and Σ_j
$l_w^{(j)}$	Parameter of axial motion ($j = c, t$)
$\mathbf{M}_{ij}$	Coordinate transformation matrix (from S_j to S_i)
$\mathbf{N}_r$	Normal vector to generating surface Σ_r ($r = c, t$)
$\mathbf{n}_f^{(i)}$	Unit normal vector to surface Σ_i in coordinate system S_f
p	Screw parameter
r_i	Radius of pitch circle of gears
r_{wp}	Radius of pitch circle of worm
$\mathbf{r}_i$,$\mathbf{r}_i^*$	Position vector of surface Σ_i, Σ_i^*
$\mathbf{r}_f^{(i)}$	Position vector of surface Σ_i in S_f
s,s_r	Displacement of rack-cutter Σ_r
$u_w^{(j)}$,$\psi_w^{(j)}$	Worm surface parameters ($j = c, t$)
$\mathbf{v}^{(ij)}$	Relative velocity of surface Σ_i point with respect to surface Σ_j point
α_n	Normal pressure angle for the nominal value of center distance
$\gamma_w^{(j)}$	Crossing angle between worm and pinion (gear) rotation axes
ΔE	Change of center distance
$\Delta\lambda_i$	Change of lead angle on the pitch circle ($i = p, g$)
$\Delta\gamma_x$,$\Delta\gamma_y$	Misalignment angle formed by crossed and intersected gear axes
$\Delta\phi_2$, $\Delta\psi_2$	Transmission error
$\lambda_i(\beta_0)$	Lead(helix) angle on pitch circle ($i = p, g$)
λ_w	Worm lead angle on pitch circle
ρ_r	Radius of circular arc of rack-cutter Σ_r in normal section ($r = c, t$)
ϕ_i,ψ_i	Rotation angle of gear i ($i = 1, 2, p, g$)
ϕ_w,$\psi_w^{(j)}$	Rotation angle of worm($j = c, t$)

1. INTRODUCTION

In the study to be conducted in this paper two types of helical gears that transform rotation between parallel axes are considered: (i) double circular arc helical gears(Novikov-Wildhaber gears) with modified topology, and (ii) involute helical gears with modified topology. An approach for the design and generation of both types of helical gears is proposed in this paper. This approach enables one to reduce the level of noise, avoid edge contact, and provide a stable bearing contact.

The circular arc helical gears (N.-W.) have been proposed by Novikov [1] and Wildhaber [2]. However, there is a significant difference between the ideas proposed by the above inventors. Wildhaber's idea is based on generation of the gears by the *same* imaginary rack-cutter that provides conjugate gear tooth surfaces being in *line* contact at every instant. Novikov proposed the application of *two mismatched* imaginary rack-cutters that provide conjugate gear tooth surfaces being in *point* contact at every instant. The great advantage of Novikov's invention is the possibility to obtain a small value of the relative normal curvature and reduce substantially the contact stresses. The weak point of Novikov's idea was the high value of bending stresses since the gear tooth surfaces are in point contact at every instant. The successful manufacturing of N.-W. gears has been accomplished by application of two mating hobs based on the idea of two mating imaginary rack-cutters. This idea has been proposed by Kudrjavtsev [3] in the former USSR and Winter and Looman [4] in Germany.

Circular arc helical gears are only a particular case of a general type of helical gears which can transform rotation with constant gear ratio and are in point contact at every instant. Litvin [5] and Davidov [6] simultaneously and independently proposed a method of generation for helical gears by "two rigidly connected" tool surfaces. According to this idea, the generating surfaces may be rack-cutter surfaces, particularly.

The kinematics of single circular arc helical gears was the subject of the paper by Litvin and C.-B. Tsay [7].

A substantial step forward in the design of N.-W. gears was the development of double circular arc helical gears with two zones of meshing. Such gears have been proposed in the former USSR [8] and the People's Republic of China [9]. The geometry of such gears was discussed in [10]. The main advantage of this development is the possibility to reduce the bending stresses keeping the advantage of reduced contact stresses.

A great disadvantage of N.-W. gears, even with two zones of contact, is their noise. The investigation performed by the authors of this paper shows that the noise results from the unfavorable shape of the function of transmission errors of misaligned gear drives (fig. 1). The function of such transmission errors is piecewise, almost linear, and has the frequency of a cycle of meshing of one pair of teeth. These transmission errors cause high vibration and noise, and therefore such transmission errors must be avoided.

Conventional helical involute gears are designed for transformation of rotation between parallel axes. Theoretically, the gear tooth surfaces are in line tangency at every instant, along a straight line that is a tangent to the helix on the gear base cylinder. However, the line contact of gear tooth surfaces can be realized only for an ideal gear drive. In reality, the crossing or intersection of axes of rotation (instead of being parallel) and errors of lead angle result in the so-called edge contact, a specific instantaneous point contact caused by curve-to-surface tangency. Here, the curve is the edge of the tooth surface of one of the mating gears and the surface is the tooth surface of the other one.

In trying to avoid edge contact, manufacturers of helical gears used various methods of crowning (deviation) of the gear tooth surfaces. However, the methods of crowning applied have not been complemented with the analysis of transmission errors caused by misalignment. The investigation conducted in this study shows that improper crowning may allow edge contact to be avoided, but cannot avoid the appearance of transmission errors of the shape shown in fig. 1(b).

In this paper a modified topology of low-noise N.-W. gears and involute helical gears are

proposed that satisfy the following requirements:

(1) The noise and vibration of both types of gears can be reduced substantially by application of a predesigned function of transmission errors of a parabolic type [11-13]. It was shown that such a predesigned function can absorb an almost linear function of transmission error, such as shown in fig. 1(b), caused by misalignment.

(2) The bearing contact is localized. Theoretically, the tooth surfaces of N.-W. gears are in tangency at every instant at two points without misalignment and at one point when the gears are misaligned. However, the two-zone contact is restored under a load due to the deflection of gear teeth. The tooth surfaces of modified helical gears are in point contact at every instant instead of line contact. The instantaneous contact of gear tooth surfaces at a point is spread over an elliptical area due to elastic deformation of the gear teeth. The dimensions of the instantaneous contact ellipse can be controlled in both cases by choosing proper design parameters.

(3) The proposed gear tooth surfaces can be ground (or cut) by a worm (hob) designed for generation of the pinion (gear). For the manufacture of the gear, the relation between the rotational motions of the gear and cutting tool is nonlinear. This can be accomplished by the application of a Computer Numerically Controlled (CNC) machine such as a Reishauer machine [14]. For the pinion, conventional manufacturing machines can be used since the relation between the rotational mations of the pinion and the cutting tool is linear.

The developed approach is based on the following ideas:

(1) Two imaginary rigidly connected rack-cutters for conjugation of gear tooth surfaces with the new topology are applied. The generated gear tooth surfaces are in point contact, and a parabolic function of transmission errors is provided.

(2) The real manufacturing of pinion-gear tooth surfaces is accomplished by a grinding worm (hob). The worm surface is an envelope to the family of surfaces of the imaginary rack-cutter. The pinion (gear) tooth surface is an envelope to the two parameter family of

worm surfaces.

(3) The meshing and contact of pinion-gear tooth surfaces of a misaligned gear drive are computerized and the influence of assembly errors is investigated. An analytical approach for determination of transmission errors caused by misalignment will be described.

2. INTERACTION OF PARABOLIC AND LINEAR FUNCTIONS OF TRANSMISSION ERRORS

Ideal gears transform rotation with constant gear ratio $m_{21} = \dfrac{N_1}{N_2}$ and the ideal transmission function is

$$\phi_2^o(\phi_1) = \frac{N_1}{N_2}\phi_1 \tag{1}$$

where N_1 and N_2 are the numbers of gear teeth.

However, the crossing or intersection of gear axes (instead of being parallel), and errors of lead angle cause a transmission function $\phi_2(\phi_1)$ that is shown in fig. 1(a). In the investigation to be presented (see sections 7 and 8) is that the function of transmission errors caused by the errors of misalignment mentioned above is a piecewise, almost linear function of transmission errors $\Delta\phi_2(\phi_1)$ with the frequency of a cycle of meshing for one pair of teeth (fig. 1(b)).

Here:

$$\Delta\phi_2(\phi_1) = \phi_2(\phi_1) - \frac{N_1}{N_2}\phi_1 \tag{2}$$

Transmission errors of this type cause a discontinuity of the driven gear angular velocity at transfer points (when one pair of teeth is changed to another one), and vibration and noise become inevitable.

It has been shown [11-13] that a predesigned parabolic function of transmission errors (fig. 2) interacting with a linear function will keep to be a parabolic function with the same parabola parameter. A parabolic function of transmission errors is much more preferable

than a linear function since the transmission function of the driven gear will be a continuous one and the stroke at the transfer point will be reduced substantially.

3. LOCALIZATION OF BEARING CONTACT

The principle of localization of the bearing contact is explained with the imaginary process for generation of helical gears by two rigidly connected rack-cutters. This principle will be applied separately for N.-W. gears and modified involute helical gears.

Generation of N.-W. Gears by Two Rack-Cutters

The imaginary process of generation of conjugate tooth surfaces is based on application of two rack-cutters that are provided by two mismatched cylindrical surfaces Σ_t and Σ_c as shown in fig. 3(a). The rack-cutter surfaces Σ_t and Σ_c are rigidly connected to each other in the process of imaginary generation, and they are in tangency along two parallel straight lines, $a-a$ and $b-b$. These lines and the parallel axes of the gears form angle β_0, that is equal to the helix angle on the pinion (gear) pitch cylinder. The normal sections of the rack-cutters have been standardized in China [9] (fig. 4(a)) and in the former USSR (fig. 4(b)) [8]. Rack-cutter surface Σ_c generates the pinion tooth surface Σ_p, and rack-cutter surface Σ_t generates the gear tooth surface Σ_g.

It is obvious that due to the mismatch of the surfaces of the two rack-cutters that generate the pinion and the gear, the tooth surfaces of the pinion and the gear will be in point contact at every instant. Each rack-cutter has two generating surfaces, above and below plane Π(fig. 3). Therefore, the pinion and the gear will have two working surfaces, and two zones of point contact.

Generation of Modified Involute Helical Gears by Two Rack-Cutters

Two imaginary rack-cutters, t and c, for generation of pinion and gear tooth surfaces,

respectively, are applied in this case as well. The rack-cutter t that generates the gear tooth surface is provided by plane Σ_t, and rack-cutter c designed for the generation of the pinion is provided by cylindrical surface Σ_c that differs slightly from plane Σ_t(fig. 5(b)). The rack-cutter surfaces Σ_t and Σ_c are rigidly connected each to other in the process of the imaginary generation, and they are in tangency along a straight line that is parallel to axis z_a and passes through point N(figs. 5(a) and (b)). This line and the parallel axes of the gears form angle β_0, that is equal to the helix angle on the pinion (gear) pitch cylinder. The normal sections of the rack-cutters are shown in figs. 5(b) and (c)). The generated pinion-gear tooth surfaces are in point contact at every instant, and there is only one zone of meshing. (Recall that N.-W. gears have two zones of meshing.)

4. GENERATION OF CONJUGATE PINION AND GEAR TOOTH SURFACES BY TWO IMAGINARY RACK-CUTTERS

Schematic of Generation

Coordinate systems used for the process of generation of the pinion(gear) tooth surfaces by rack-cutters are shown in fig. 6. The fixed coordinate systems S_m and S_n are rigidly connected to the frame of the cutting machines that are used for generation of the pinion and the gear, respectively. Movable coordinate systems $S_r(r = c, t)$, S_p and S_g are rigidly connected to the rack-cutters, the pinion and the gear, respectively.

The rack-cutter surface Σ_r $(r = t,\ c)$ is represented in S_r by the equation

$$\mathbf{r}_r = \mathbf{r}_r(\theta_r, u_r) \tag{3}$$

where u_r and θ_r are the surface parameters.

The unit normal to the rack-cutter surface is represented as

$$\mathbf{n}_r = \frac{\mathbf{N}_r}{|\mathbf{N}_r|}, \qquad \mathbf{N}_r = \frac{\partial \mathbf{r}_r}{\partial \theta_r} \times \frac{\partial \mathbf{r}_r}{\partial u_r} \tag{4}$$

Surfaces Σ_t and Σ_c of the rack-cutters have the same direction and orientation of teeth determined by β_0.

The installment of the rack-cutters shown in fig. 6 provide: (i) direction of pinion teeth that is opposite to the direction of teeth of rack-cutter c (fig. 6(a)), and (ii) direction of gear teeth that is the same as of rack-cutter t (fig. 6(b)).

Generation of Pinion-Gear Tooth Surfaces

In the process for generation, the two rigidly connected rack-cutters perform translational motion while the pinion and the gear perform rotational motions as shown in fig. 6. To provide a predesigned parabolic function of transmission errors for each cycle of meshing, it is necessary to observe certain relations between the motions of the rack-cutters and the gears, respectively.

The angle ψ_p of pinion rotation and the displacement s_c of rack-cutter Σ_c are related by the following linear function

$$\psi_p = \frac{s_c}{r_p} \tag{5}$$

Here: r_p is the radius of the pinion pitch cylinder.

The angle ψ_g of gear rotation and the displacement s_t of rack-cutter Σ_t are related as follows

$$\psi_g = \frac{N_p}{N_g}\left(\frac{s_t}{r_p}\right) - a\left(\frac{s_t}{r_p}\right)^2 \tag{6}$$

Here: N_p and N_g are the numbers of the pinion and gear teeth, respectively.

The generated surface Σ_i (the pinion or the gear tooth surface) is represented by the family of lines of contact between the rack-cutter surface Σ_r ($r = c, t$) and the surface Σ_i ($i = p, g$) of the pinion (gear) being generated. Surface Σ_i is represented by the following equations

$$\mathbf{r}_i(u_r, \theta_r, \psi_i) = \mathbf{M}_{ir}(\psi_i)\ \mathbf{r}_r(u_r, \theta_r) \tag{7}$$

$$\frac{\partial \mathbf{r}_i}{\partial \psi_i} \cdot \left(\frac{\partial \mathbf{r}_i}{\partial \theta_r} \times \frac{\partial \mathbf{r}_i}{\partial u_r}\right) = f(u_r, \theta_r, \psi_i) = 0 \tag{8}$$

Here: $i = p$ while $r = c$; $i = g$ while $r = t$. Equation (8) is the equation of meshing [13]. An alternative and more simple way of derivation of the equation of meshing is as follows [13]:

$$\mathbf{N}_r \cdot \mathbf{v}_r^{(ri)} = f(u_r, \theta_r, \psi_i) = 0 \tag{9}$$

Here: $\mathbf{N}_r$ is the normal to the rack-cutter surface; $\mathbf{v}_r^{(ri)}$ is the relative velocity in the process of meshing. Vectors $\mathbf{N}_r$ and $\mathbf{v}_r^{(ri)}$ can be represented in coordinate system S_r rigidly connected to the rack-cutter.

According to the relation of motions by equations (5) and (6), the rack-cutter c generates the pinion tooth surface as a helicoid. The gear tooth surface generated by rack-cutter t is modified, and it is not a helicoid.

5. GENERATION OF CONJUGATE PINION AND GEAR TOOTH SURFACES BY WORMS

Introduction. The real generation of pinion-gear tooth surfaces (by cutting or grinding) is preferable if based on application of a worm, especially in the case of grinding. Grinding of double circular arc helical gears and modified involute helical gears by worms can be accomplished by application of Reishauer CNC gear grinding machines [11]. The grinding worm must be provided with the required thread surface (see below). During the process for generation, the worm and the pinion (gear) being generated must perform related rotational motions, and, in addition, the worm (or the pinion (gear)) must perform translational motion

(feed motion) in the direction of the axis of the pinion (gear). The feed motion must be provided since the pinion (gear) tooth surface and the worm thread surface are in point contact at every instant. The relations between the rotational motions of the worm and the pinion are linear, but nonlinear in the case for generation of the gear since a predesigned parabolic function of transmission errors must be provided. This is the reason why a CNC (computer numerical control) machine is required for the generation of the gear.

Basic Concepts. (1) The worm thread surface is the *envelope* to the family of rack-cutter surfaces. In some studies the determination of the thread surface of the generating worm is based on the following considerations: (a) it is assumed that the normal section L_n of the worm designated as L_n is the same as the normal section of the rack-cutter; (b) the thread surface is generated by the screw motion of L_n about the worm axis. However, this approach must be considered as an approximate one only, whose precision is sufficient only for a worm with a small lead angle.

(2) While a pinion (gear) is generated by a hob (grinding worm) we consider that: (a) the axes of the hob and the gear are crossed, (b) the hob (grinding worm) and the gear being generated perform related rotations about their axes, and (c) the hob performs in addition to rotation the translational motion in the direction of the axis of the pinion(gear) that is called the feed motion.

(3) The rack-cutter surface Σ_r and the pinion (gear) tooth surface Σ_i $(i = p, g)$ are in line tangency at every instant, along line L_{ri}. The rack-cutter surface Σ_r and the worm thread surface Σ_w are as well in line tangency at every instant, along line L_{rw}. Lines L_{ri} and L_{rw} do not coincide but intersect each other at every instant at a point. This means that surface Σ_w and Σ_i are in *point* tangency at every instant, and the generation of Σ_i by Σ_w requires the feed motion of the worm.

There are two alternative methods for determination of the equations of the pinion(gear) tooth surface Σ_i $(i = p, g)$: (i) as the envelope to the rack-cutter surfaces Σ_r $(r = c, t)$, or

(ii) as the envelope to the two parameter family of surfaces of the worm. Both approaches provide the same pinion (gear) tooth surface.

6. COMPUTERIZED SIMULATION OF MESHING AND CONTACT

The computerized simulation of meshing is based on the equations that provide continuous tangency of pinion and gear tooth surfaces. The simulation can be accomplished for aligned and misaligned gear drives. The computerized simulation of contact is based on determination of the contact ellipse at each instant.

Three coordinate systems, S_p, S_g and S_f are applied for investigation(fig. 7). The fixed coordinate system S_f is rigidly connected to the housing of the gear drive (fig. 7(a)). The movable coordinate systems S_p and S_g are rigidly connected to the pinion and the gear, respectively. An auxiliary coordinate system S_q is applied for simulation of meshing when the gear axis is crossed or intersected with the pinion axis instead of being parallel, and when the shortest distance between the pinion and gear axes is changed. The misalignment angle $\Delta\gamma$ is decomposed into two components, $\Delta\gamma_x$ and $\Delta\gamma_y$ that represent the crossing angle and the intersection angle, respectively. Fig. 7(b) and 7(c) show the orientation of coordinate system S_q with respect to S_f when the axes of rotation of the gear and the pinion are crossed or intersected, respectively. The pinion performs rotational motion about the z_f-axis. The axis of gear rotation is z_q. The shortest distance between the axes of rotation is designated as E_{pg}.

We represent the pinion and gear tooth surfaces, Σ_p and Σ_g, and their unit normals in coordinate system S_f. Then we use the conditions of continuous tangency of the tooth surfaces and simulate as well the gear misalignment.

The determination of dimensions and orientation of the instantaneous contact ellipse requires the knowledge of the principal curvatures and directions of the contacting surfaces and the elastic approach of surfaces. This problem can be substantially simplified if the pinion

and gear principal curvatures and directions are expressed in terms of the principal curvatures and directions of the generating surfaces and parameters of motion [12,13]. The output of TCA are [12,13]: (i) the paths of contact on gear tooth surfaces, (ii) the transmission errors, and (iii) the bearing contact formed by the instantaneous contact ellipses.

The simulation of meshing and contact has been performed for N.-W. gears and the modified involute helical gears in the examples of section 8 to follow. The main results of TCA are as follows:

(i) The linear finctions of transmission errors caused by misalignment are absorbed indeed by the predesigned parabolic function of transmission errors. The advantage of such absorption is the reduction of noise and vibration.

(ii) The bearing contact for modified involute helical gears is stable, and the instantaneous contact ellipse moves along but not across the tooth surface. We can expect that this will benefit the conditions of lubrication.

(iii) The path of contact on the tooth surface is a helix in the case of an aligned gear drive, and almost a helix for a misaligned gear drive.

(iv) Theoretically, the contact ratio for an unloaded gear drive is equal to one due to the existence of transmission errors. However, the contact ratio under load is increased due to the deflection of teeth.

7. ANALYTICAL DETERMINATION OF TRANSMISSION ERRORS CAUSED BY MISALIGNMENT

While computerized analysis of meshing enables one to determine the transmission errors numerically. Our goals here are: (i) to provide analytical solutions, (ii) to prove that the induced function of transmission errors is almost a linear one with respect to the rotation angle ϕ_1 of the pinion(driving gear), and (iii) represent this function in terms of the errors of angular alignment and the gear design parameters. We apply for the solution the approach

proposed in [10,12,13] that is based on the following considerations.

Assume that the tooth surfaces of an aligned gear drive are in tangency at a current point of the line of action. This line for the helical gears discussed above is almost a straight line that is parallel to the gear axes. Due to misalignment, the point of tangency of the theoretical tooth surfaces is displaced, and the surfaces interfere each other or a backlash occurs. To restore the tooth surface contact, it is sufficient to provide a compensating turn of one of the mating gears, say gear 2. Angle $|\boldsymbol{\Delta\phi_2}|$ of the compensating turn can be determined by using the equation

$$(\boldsymbol{\Delta\phi}_2 \times \mathbf{r}_2 + \Delta\mathbf{q}) \cdot \mathbf{n} = 0 \tag{10}$$

Here: $\boldsymbol{\Delta\phi}_2$ is the vector of the compensating angle of rotation of gear 2; $\mathbf{n}$ is the unit normal at the contact point; $\mathbf{r}_2$ is the position vector of the current point of the line of action; $\Delta\mathbf{q}$ is the displacement of the contact point caused by misalignment.

Determination of Linear Functions of Transmission Errors

Three types of angular misalignment are considered: crossing of axes, intersection of axes, and error of the lead angle of the pinion(or the gear). Using equation (10), we have determined the following equation of transmission errors for both types of helical gears considered above.

$$\Delta\phi_2 = b\phi_1 \tag{11}$$

where,

$$b = -\frac{N_p}{N_g}\tan\lambda_p(\Delta\gamma_y + \frac{\tan\alpha_n}{\sin\lambda_p}\Delta\gamma_x - \frac{\Delta\lambda_p}{\sin^2\lambda_p} + \frac{\Delta\lambda_g}{\sin^2\lambda_g}) \tag{12}$$

Here: $\Delta\gamma_x$ is the crossing angle (fig. 7(b)); $\Delta\gamma_y$ is the intersection angle (fig. 7(c)); $\Delta\lambda_p$ and $\Delta\lambda_g$ are the errors of the lead angles of the pinion and the gear, respectively.

Influence of Change of Center Distance

The change of center distance of N.-W. gears and modified involute helical gears does not cause transmission errors but only the shift of the bearing contact (the path of contact). The shift can be evaluated as the change of the pressure angle determined as follows:

(i) In the case of N.-W. gears we have [10,16]

$$\sin\alpha_n^* = \frac{\Delta E - y_{ot} + y_{oc}}{\rho_t - \rho_c} \tag{13}$$

where $y_{or}(r = c, t)$ is the coordinate of the circle center corresponding to circular arc $\rho_r(r = c, t)$(fig. 4); α_n^* is the pressure angle in the normal section; $\alpha_n^* = \alpha_n$, where α_n is the nominal value of the pressure angle, if ΔE, the change of the center distance, is equal to zero. The difference $(\alpha_n^* - \alpha_n)$ indicates the shift of the path of contact (the bearing contact) on the tooth surface.

(ii) The influence of the center distance change, in the case of modified involute helical gears, is represented by the equation

$$\sin\alpha_t^* = (\sin^2\alpha_t + \frac{2\Delta E}{E}\cos\alpha_t)^{0.5} \tag{14}$$

where α_t^* and α_t represent the transverse pressure angles for the center distances $(E+\Delta E)$ and E, respectively.

8. NUMERICAL EXAMPLES

The theory and approach developed in this paper are illustrated with two numerical examples: (i) the double circular arc gear drive and (ii) the modified involute helical gear drive. The computations have been performed by application of TCA (Tooth Contact Analysis) computer programs.

Example 1, Double Circular Arc Gear Drive

The input design parameters used in this example are: $P_n = 10\ \frac{1}{in.}$ (module $m_n = 2.54mm$), $N_p = 12$, $N_g = 94$, $\alpha_o = 27°$, $\beta_0 = 30°$, $L = 33mm$, $a = 0.00053$, $\delta = 0.001\ mm.$, where δ is the elastic approach.

Aligned Gear Drive. Contact paths and contact ellipses on the surfaces for a single tooth are shown in fig. 8 for the ideal case. Two contact points on the surface of a single tooth exist only in the part of the area of meshing. Two contact points existing simultaneously are shown by circles on the paths of contact (fig. 9). The transmission errors are determined by a predesigned parabolic function with the maximal value of 8 arc seconds.

Influence of Misalignment. The misalignment has been simulated as the change $\Delta\gamma$ of orientation of gear axes, when the axes become crossed or intersected instead of being parallel, and by the change $\Delta\lambda$ of the lead angle. There is only one instantaneous contact point of gear tooth surfaces Σ_p and Σ_g: (i) contact point $M^{(b)}$ that is located on the lower part of the tooth surface if the errors ($\Delta\gamma_x$, $\Delta\gamma_y$ and $\Delta\lambda$) are positive, and (ii) contact point $M^{(a)}$ that is located on the upper surface if the errors above are negative. Surfaces Σ_p and Σ_g at the second theoretical contact point are separated. However, the instantaneous contact of surfaces at two points may be restored due to lapping or wearing of the surfaces under the load.

The results of TCA for various errors of alignment are shown in Table 1. The results show that the predesigned parabolic function indeed absorbs indeed the linear functions of transmission errors caused by misalignment error $\Delta\gamma$ and the error of lead angle $\Delta\lambda$, and keeps the same slope; the maximal transmission error is 8 arc seconds.

Influence of Change ΔE of Center Distance. The results of computation are: $\alpha_n^* = 22.0419°$ when $\Delta E = 0.03\ mm$; and $32.1911°$ when $\Delta E = -0.03\ mm$. This means that the bearing contact is shifted up and down depending on the sign of ΔE. The conditions

of meshing are the same as shown in fig. 9 since only ΔE, but not $\Delta\gamma$ and $\Delta\lambda$, exist. Two contact points exist only partially during the whole cycle of meshing.

Example 2, Modified Involute Helical Gear Drive

The input design parameters used in this example are: $P_n = 5\ \frac{1}{in.}$ (module $m_n = 5.8mm$), $N_p = 20$, $N_g = 100$, $\alpha_n = 20°$, $\beta_0 = 30°$, $L = 1.6\ in.$ ($40.64mm$), $a = 0.0015$, $\delta = 0.0003\ in(0.007\ mm)$.

Aligned Gear Drive. Figures 10 and 11 show the predesigned parabolic type of transmission errors and the contact pattern for the case without misalignment ($\Delta\gamma_x = \Delta\gamma_y = \Delta\lambda_p = 0$, $\Delta E = 0$). The maximal transmission error is 8 arc seconds.

Influence of Misalignment. The results of investigation of the influence of misalignment (Table 2) show: (a) the predesigned parabolic function of transmission errors indeed absorbs the linear function of transmission errors caused by misalignment, and (b) the contact paths on the pinion- gear tooth surfaces are located in the neighborhood of the ideal contact paths.

Influence of Change ΔE of Center Distance. The transverse pressure angle is slightly changed due to the change of the center distance: $\alpha_t^* = 22.84°$ when $\Delta E = 0.1\ mm.$; and $\alpha_t^* = 22.76°$ when $\Delta E = -0.1\ mm.$ ($\alpha_t = 22.80°$ when $\Delta E = 0$).

9. CONCLUSION

Based on the results contained in this study, the following conclusions can be made:

(1) The absorption of a linear function of transmission errors by a predesigned parabolic function has been confirmed.

(2) An approach for localization of the bearing contact for helical gears has been developed.

(3) Conjugation of gear tooth surfaces by application of two imaginary rack-cutters has been developed.

(4) Pinion and gear tooth surfaces with modified topology generated by a worm for determination of a favorable shape of transmission errors has been developed.

(5) Computerized simulation of meshing and contact has been investigated.

(6) Analytical determination of functions of transmission errors caused by misalignment has been developed.

(7) Two numerical examples of N.-W. gear drive and modified involute helical gear drive for illustration of the developed theory have been provided.

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Table 1: The results of TCA for misaligned N.-W. gear drive

misalignment		without parabolic function $a = 0$		with parabolic function $a = 0.00053$	
		b	jump	$\Delta\psi_{2max}$	position errors
$\Delta\gamma_y$	$3'$	-0.00011	$12.3''$	$8''$	$-96''$
	$-3'$	0.00011	$12.3''$	$8''$	$105.5''$
$\Delta\gamma_x$	$3'$	-0.00019	$21''$	$8''$	$13''$
	$-3'$	0.00019	$21.0''$	$8''$	$2.6''$
$\Delta\lambda_p$	$3'$	0.00026	$27.8''$	$8''$	$2.6''$
	$-3'$	-0.00026	$17.8''$	$8''$	$13.0''$
$'$ — arc minute, $''$ — arc second					

Table 2: The results of TCA for involute helical misaligned gear drive

misalignment		without parabolic function $a = 0$		with parabolic function $a = 0.0015$		
		b	jump	$\Delta\psi_{2max}$	position errors	shift of contact paths
$\Delta\gamma_x$	$4'$	-0.0004	$25.9''$	$8''$	$-1.2''$	up
	$-4'$	0.0004	$25.9''$	$8''$	$3.2''$	down
$\Delta\gamma_y$	$4'$	-0.00017	$11.0''$	$8''$	$-137.8''$	up
	$-4'$	0.00017	$11.0''$	$8''$	$138.1''$	down
$\Delta\lambda_o$	$4'$	0.00054	$35.0''$	$8''$	$3.2''$	down
	$-4'$	-0.00054	$35.0''$	$8''$	$1.0''$	up
$'$ — arc minute, $''$ — arc second						

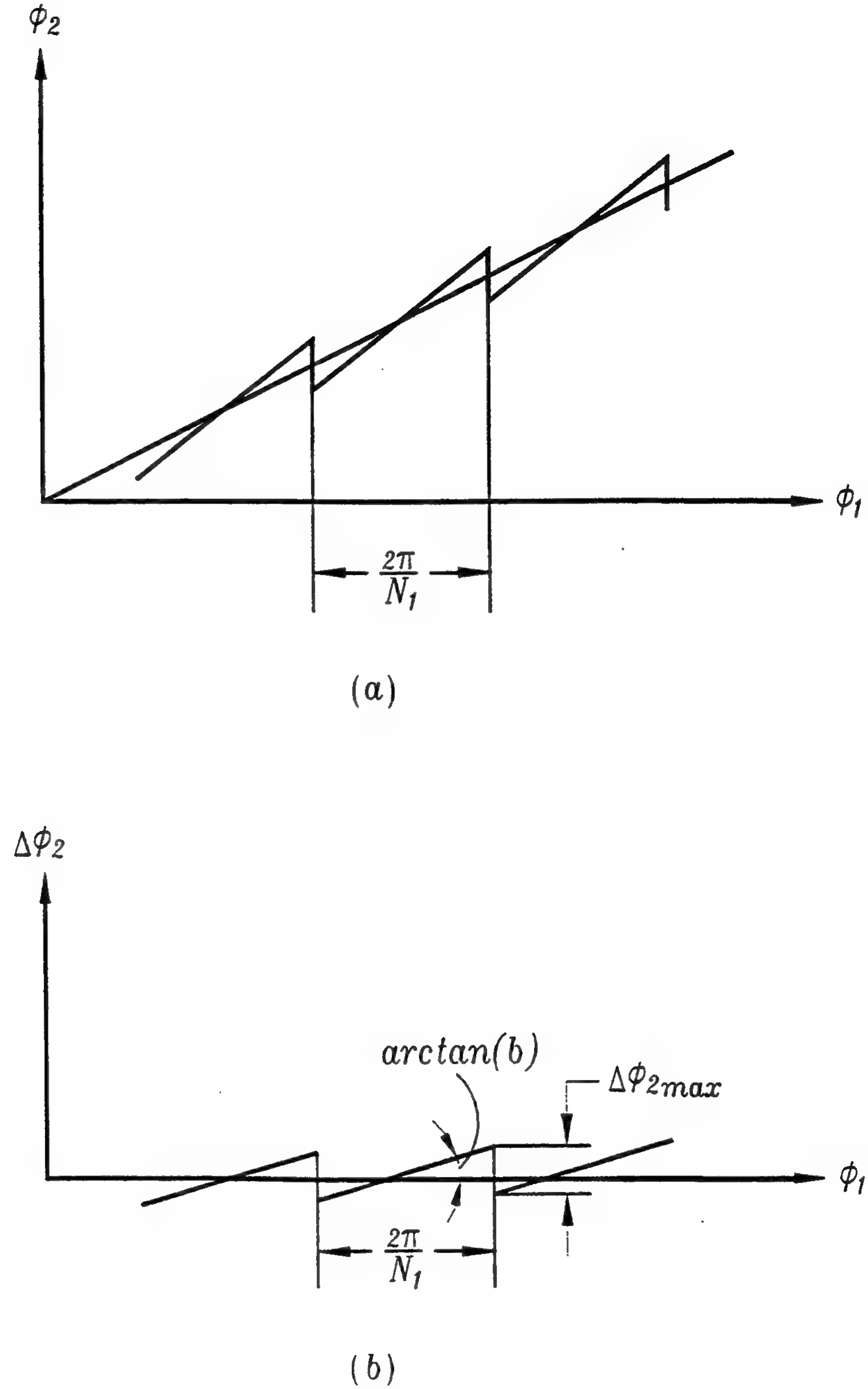


Figure 1: Transmission function and transmission errors for a misaligned gear drive

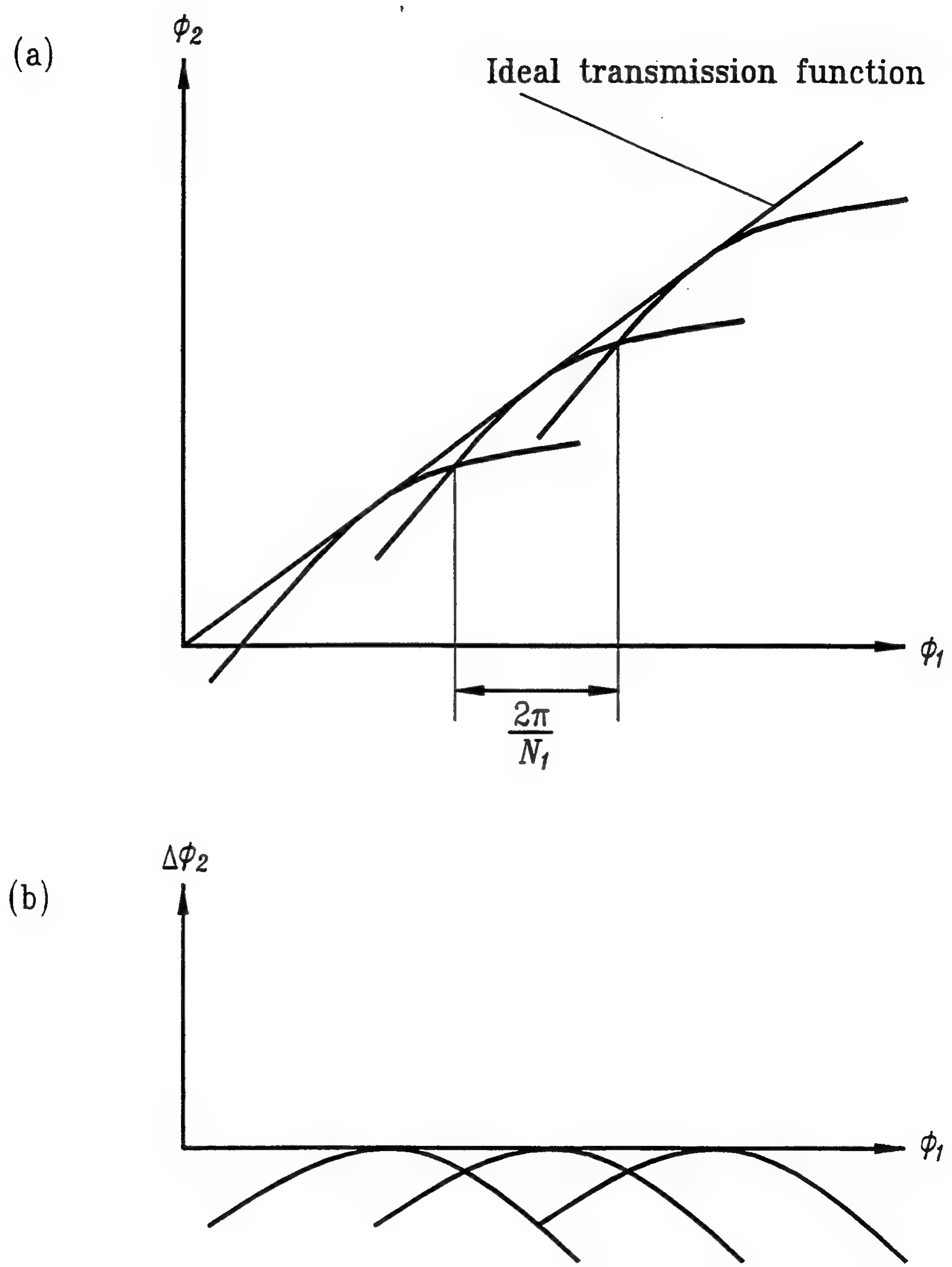


Fig. 2 Transmission function and predesigned parabolic function of transmission errors

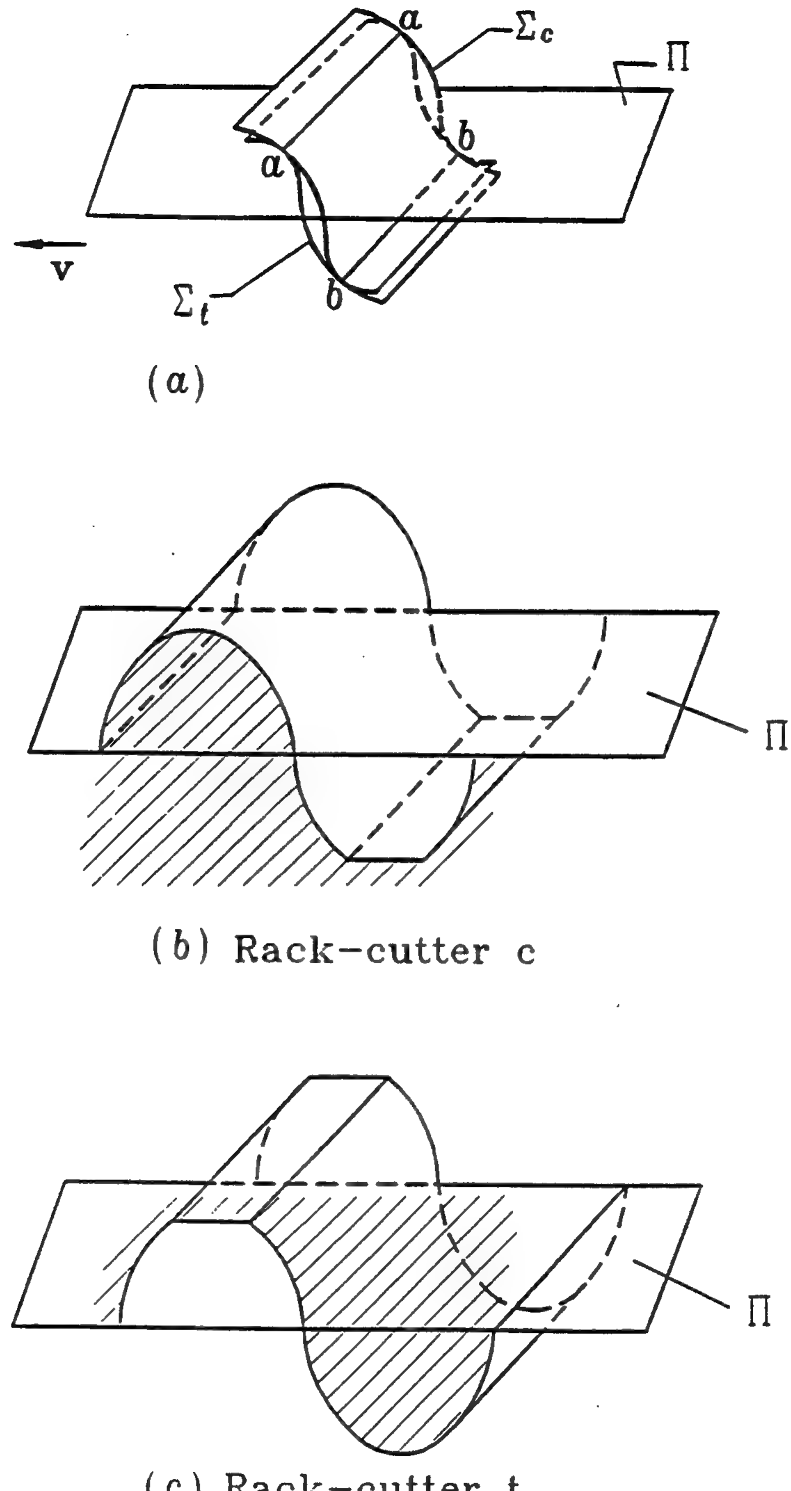
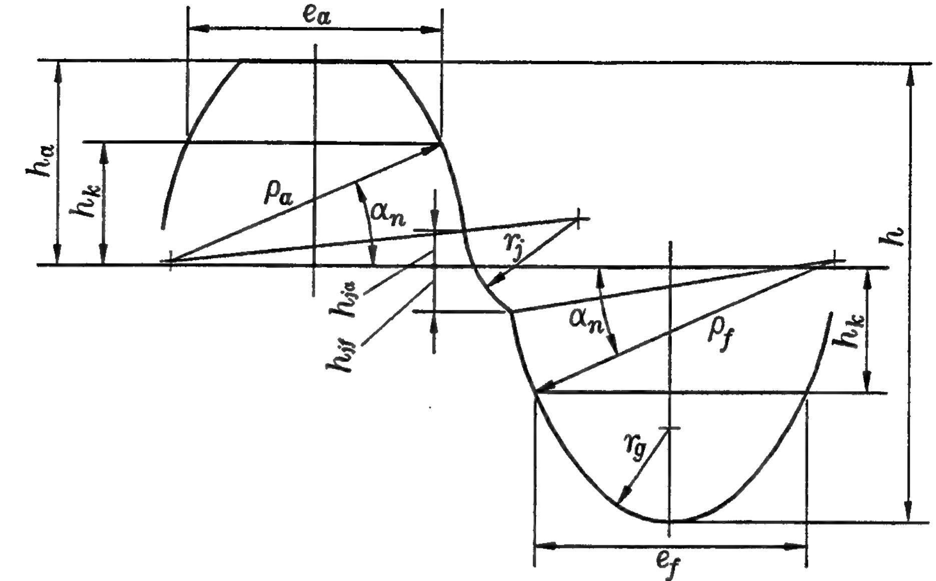
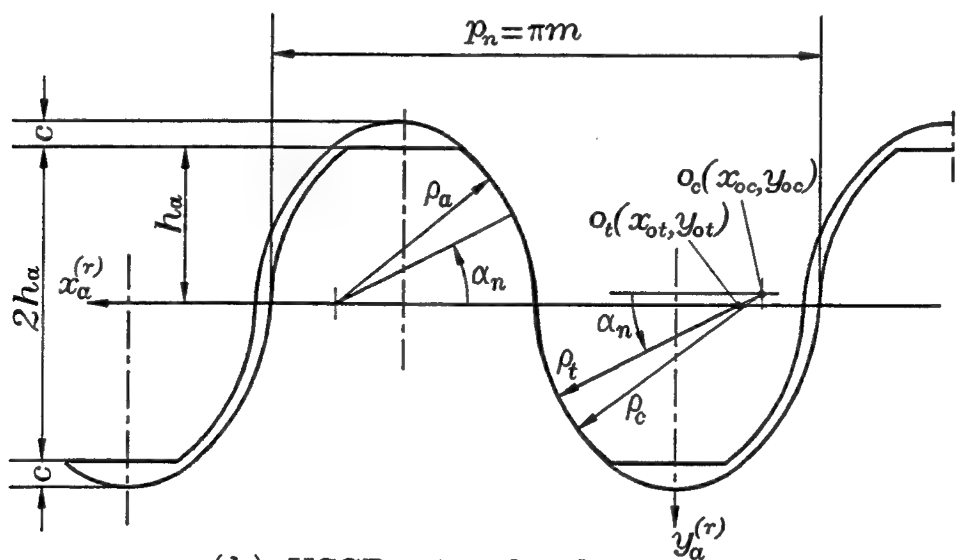


Figure 3: Surfaces of imaginary rack-cutters



(a) Chinese standard



(b) USSR standard

Figure 4: Standardized rack-cutter profiles

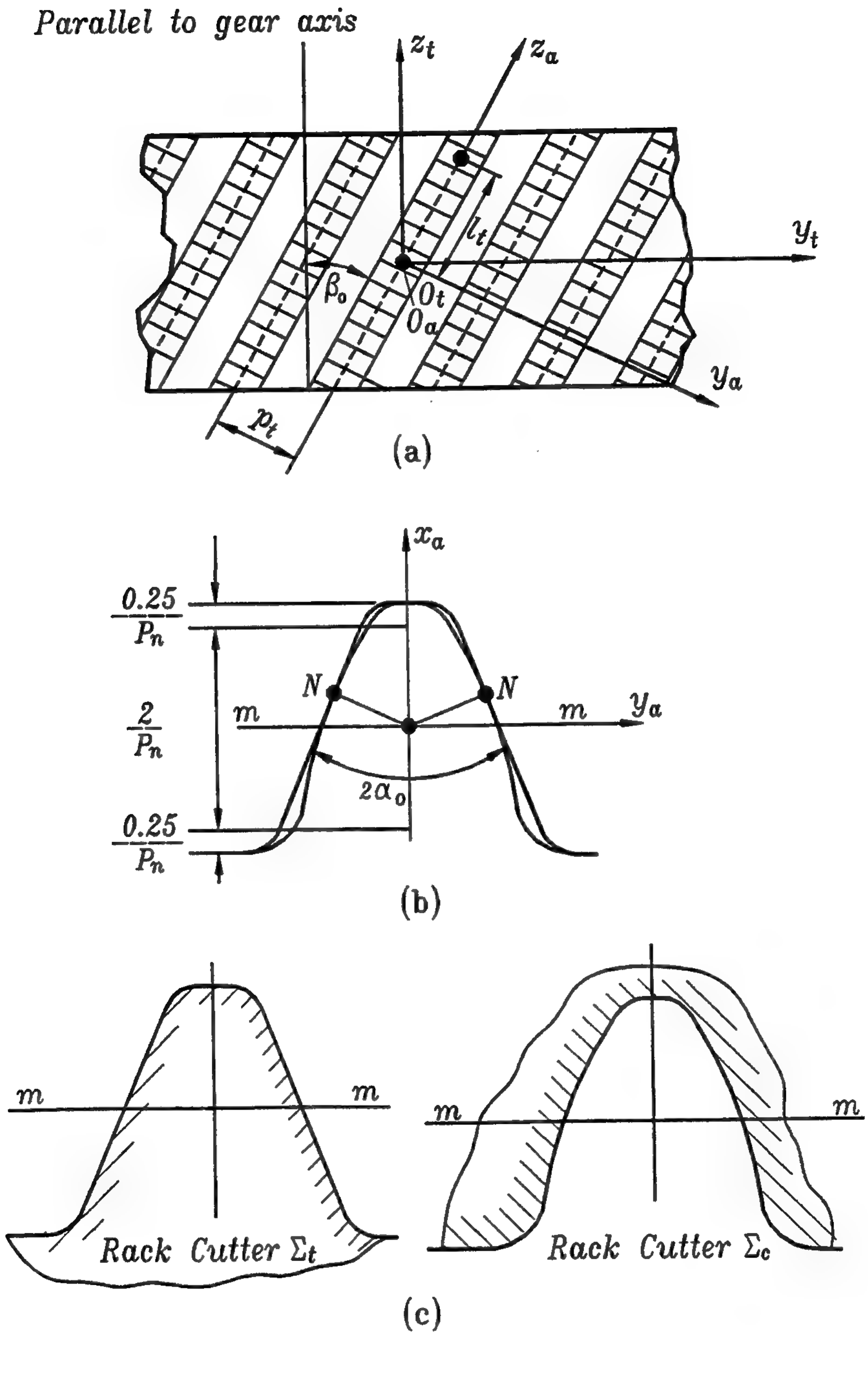


Figure 5: Schematic of rack-cutters for generation of helical involute gears

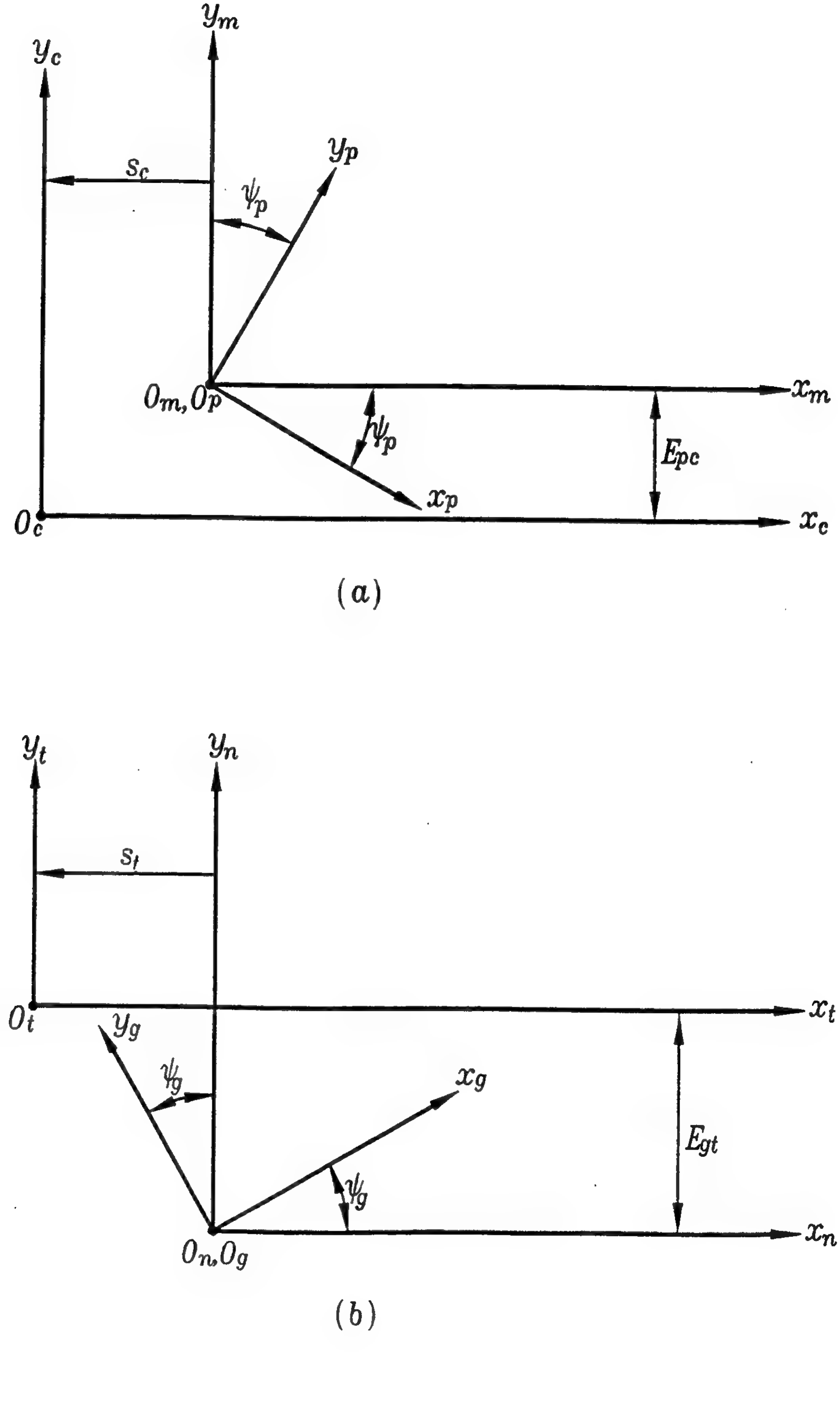


Figure 6: Generation of pinion and gear by rack-cutters

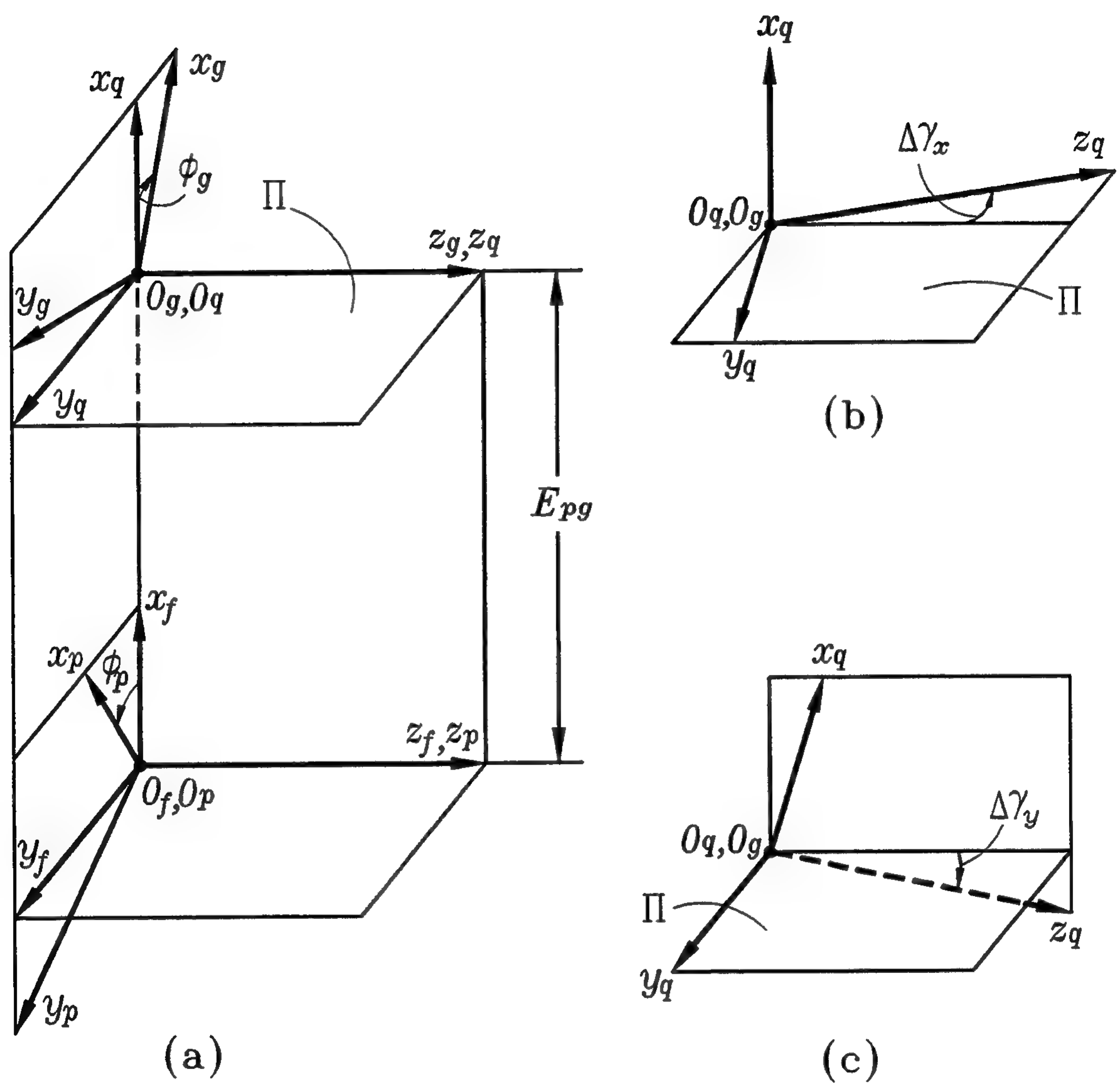


Fig. 7 Coordinate systems applied for simulation of meshing

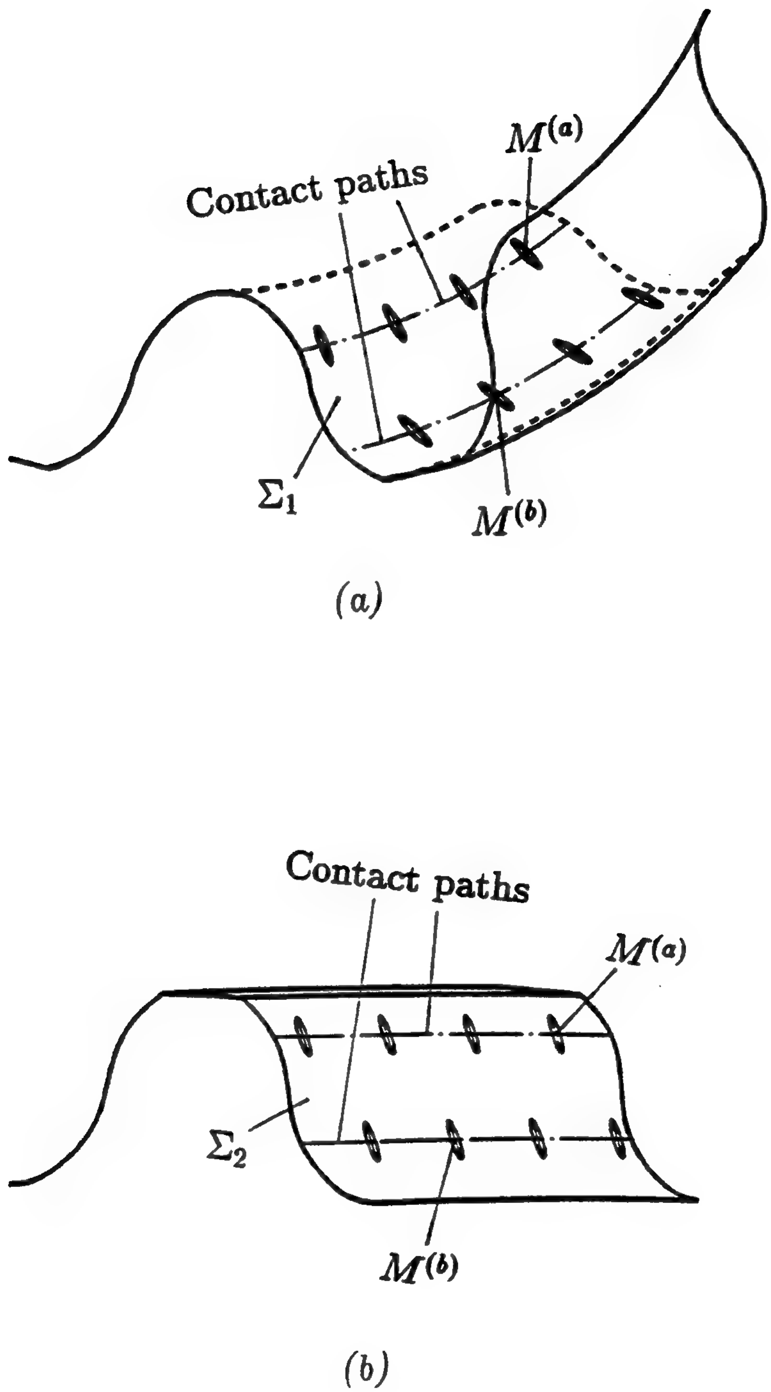
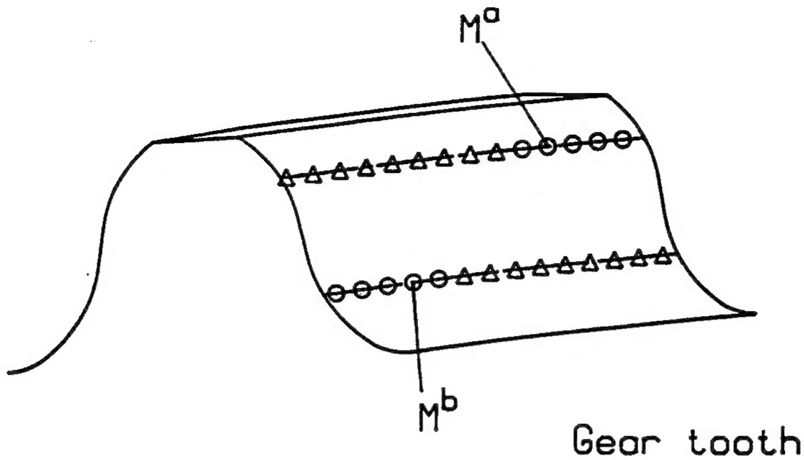
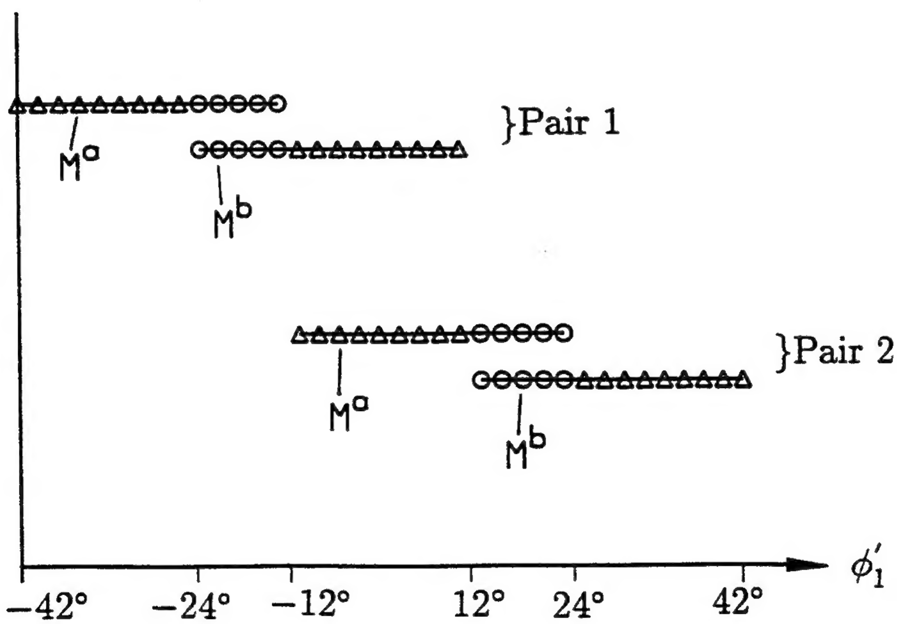


Fig. 8 Paths of contact and contact ellipses on pinion–gear tooth surfaces



(a)



(b)

Figure 9: Sequence of contact points for aligned gear drive

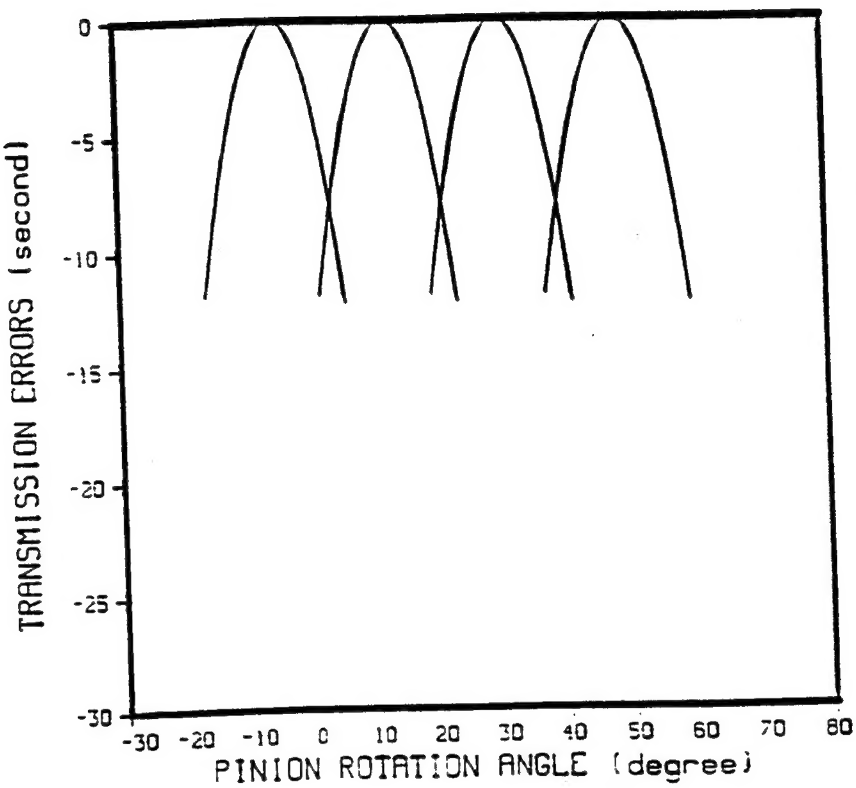


Figure 10: Transmission errors for modified involute helical gears with aligned axes

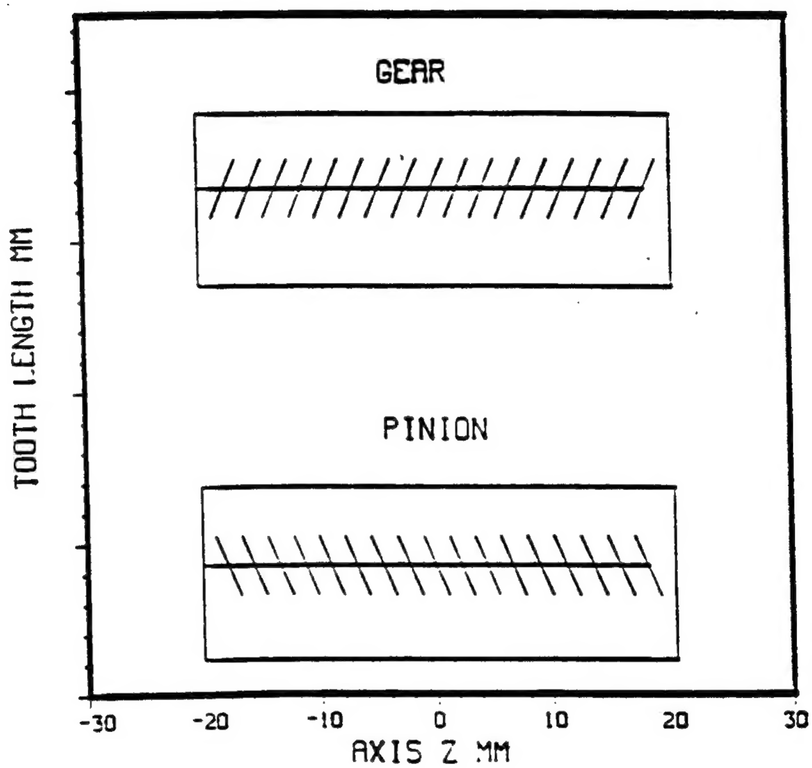


Figure 11: Contact pattern for modified involute helical gears with aligned axes

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13. ABSTRACT ***(Maximum 200 words)***

An approach for design and generation of low-noise helical gears with localized bearing contact is proposed. The approach is applied to double circular arc helical gears and modified involute helical gears. The reduction of noise and vibration is achieved by application of a predesigned parabolic function of transmission errors that is able to absorb a discontinuous linear function of transmission errors caused by misalignment. The localization of the bearing contact is achieved by the mismatch of pinion-gear tooth surfaces. Computerized simulation of meshing and contact of the designed gears demonstrated that the proposed approach will produce a pair of gears that has a parabolic transmission error function even when misalignment is present. Numerical examples for illustration of the developed approach are given.

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